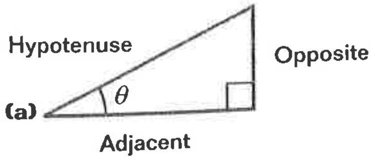
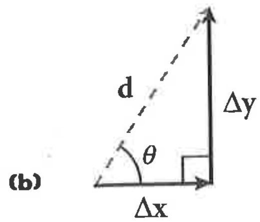
Use the tangent function to find the direction of the resultant



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$



$$\tan\theta = \frac{\Delta y}{\Delta x}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

Figure 9
(a) The tangent function can be applied to any right triangle, and **(b)** it can also be used to find the direction of a resultant displacement.

In order to completely describe the tourist's displacement, you must also know the direction of the tourist's motion. Because **Δx, Δy,** and **d** form a right triangle, as shown in **Figure 9(b),** the inverse tangent function can be used to find the angle *θ*, which denotes the direction of the tourist's displacement.

For any right triangle, the tangent of an angle is defined as the ratio of the opposite and adjacent legs with respect to a specified acute angle of a right triangle, as shown in **Figure 9(a).**

As shown below, the magnitude of the opposite leg divided by the magnitude of the adjacent leg equals the tangent of the angle.

DEFINITION OF THE TANGENT FUNCTION FOR RIGHT TRIANGLES

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \textbf{tangent of angle} = \frac{\textbf{opposite leg}}{\textbf{adjacent leg}}$$

The inverse of the tangent function, which is shown below, gives the angle.

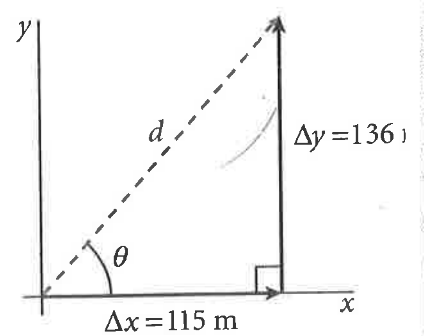
$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$$

SAMPLE PROBLEM A

Finding Resultant Magnitude and Direction

PROBLEM

An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid's height is 136 m and its width is 2.30×10^2 m. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?



SOLUTION

1. DEFINE

Given: $\Delta y = 136$ m $\qquad \Delta x = \frac{1}{2}(\text{width}) = 115$ m

Unknown: $d = ?$ $\qquad \theta = ?$

Diagram: Choose the archaeologist's starting position as the origin of the coordinate system.

2. PLAN **Choose an equation or situation:**
The Pythagorean theorem can be used to find the magnitude of the archaeologist's displacement. The direction of the displacement can be found by using the tangent function.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Rearrange the equations to isolate the unknowns:

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

3. CALCULATE **Substitute the values into the equations and solve:**

$$d = \sqrt{(115\text{ m})^2 + (136\text{ m})^2}$$

$$\boxed{d = 178\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{136\text{ m}}{115\text{ m}}\right)$$

$$\boxed{\theta = 49.8°}$$

TIP *Be sure your calculator is set to calculate angles measured in degrees. Some calculators have a button labeled "DRG" that, when pressed, toggles between degrees, radians, and grads.*

4. EVALUATE Because d is the hypotenuse, the archaeologist's displacement should be less than the sum of the height and half of the width. The angle is expected to be more than 45° because the height is greater than half of the width.

PRACTICE A

Finding Resultant Magnitude and Direction

1. A truck driver is attempting to deliver some furniture. First, he travels 8 km east, and then he turns around and travels 3 km west. Finally, he turns again and travels 12 km east to his destination.
   a. What distance has the driver traveled?
   b. What is the driver's total displacement?
2. While following the directions on a treasure map, a pirate walks 45.0 m north and then turns and walks 7.5 m east. What single straight-line displacement could the pirate have taken to reach the treasure?
3. Emily passes a soccer ball 6.0 m directly across the field to Kara. Kara then kicks the ball 14.5 m directly down the field to Luisa. What is the ball's total displacement as it travels between Emily and Luisa?
4. A hummingbird, 3.4 m above the ground, flies 1.2 m along a straight path. Upon spotting a flower below, the hummingbird drops directly downward 1.4 m to hover in front of the flower. What is the hummingbird's total displacement?

RESOLVING VECTORS INTO COMPONENTS

components of a vector

the projections of a vector along the axes of a coordinate system

In the pyramid example, the horizontal and vertical parts that add up to giv the tourist's actual displacement are called **components.** The x component parallel to the x-axis. The y component is parallel to the y-axis. Any vector ca be completely described by a set of perpendicular components.

In this textbook, components of vectors are shown as outlined, ope arrows. Components have arrowheads to indicate their direction. Compo nents are scalars (numbers), but they are signed numbers, and the direction important to determine their sign in a particular coordinate system.

You can often describe an object's motion more conveniently by breaking single vector into two components, or *resolving* the vector. Resolving a vect allows you to analyze the motion in each direction.

This point may be illustrated by examining a scene on the set of a new acti movie. For this scene, a biplane travels at 95 km/h at an angle of 20° relative the ground. Attempting to film the plane from below, a camera team travels i truck that is directly beneath the plane at all times, as shown in **Figure 10.**

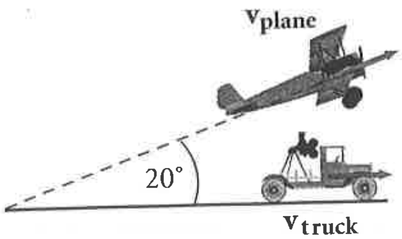


Figure 10
A truck carrying a film crew must be driven at the correct velocity to enable the crew to film the underside of a biplane. The plane flies at 95 km/h at an angle of 20° relative to the ground.

To find the velocity that the truck must maintain to stay beneath the pla we must know the horizontal component of the plane's velocity. Once mo the key to solving the problem is to recognize that a right triangle can drawn using the plane's velocity and its x and y components. The situati can then be analyzed using trigonometry.

The sine and cosine functions are defined in terms of the lengths of sides of such right triangles. The sine of an angle is the ratio of the leg op site that angle to the hypotenuse.

DEFINITION OF THE SINE FUNCTION FOR RIGHT TRIANGLES

$$\sin \theta = \frac{opp}{hyp} \qquad \text{sine of an angle} = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

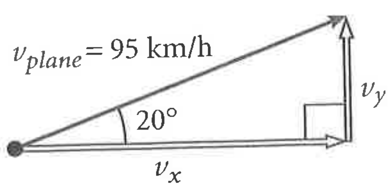


Figure 11
To stay beneath the biplane, the truck must be driven with a velocity equal to the x component (v_x) of the biplane's velocity.

In **Figure 11,** the leg opposite the 20° angle represents the y component which describes the vertical speed of the airplane. The hypotenuse, $\mathbf{v}_{plan}$ the resultant vector that describes the airplane's total velocity.

The cosine of an angle is the ratio between the leg adjacent to that a and the hypotenuse.

DEFINITION OF THE COSINE FUNCTION FOR RIGHT TRIANGLES

$$\cos \theta = \frac{adj}{hyp} \qquad \text{cosine of an angle} = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

In **Figure 11,** the adjacent leg represents the x component, v_x, w describes the airplane's horizontal speed. This x component equals the s that the truck must maintain to stay beneath the plane. Thus, the truck maintain a speed of $v_x = (\cos 20°)(95 \text{ km/h}) = 90$ km/h.

SAMPLE PROBLEM B

Resolving Vectors

PROBLEM

Find the components of the velocity of a helicopter traveling 95 km/h at an angle of 35° to the ground.

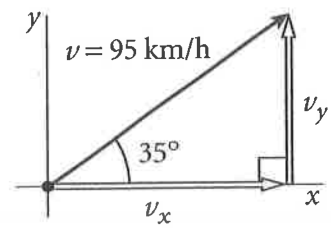
SOLUTION

1. DEFINE

Given: $v = 95$ km/h $\quad \theta = 35°$

Unknown: $v_x = ?\quad v_y = ?$

Diagram: The most convenient coordinate system is one with the x-axis directed along the ground and the y-axis directed vertically.



2. PLAN

Choose an equation or situation:
Because the axes are perpendicular, the sine and cosine functions can be used to find the components.

$$\sin\theta = \frac{v_y}{v}$$

$$\cos\theta = \frac{v_x}{v}$$

Rearrange the equations to isolate the unknowns:

$$v_y = v\sin\theta$$
$$v_x = v\cos\theta$$

> **TIP** *Don't assume that the cosine function can always be used for the x-component and the sine function can always be used for the y-component. The correct choice of function depends on where the given angle is located. Instead, always check to see which component is adjacent and which component is opposite to the given angle.*

3. CALCULATE

Substitute the values into the equations and solve:

$$v_y = (95 \text{ km/h})(\sin 35°)$$

$$\boxed{v_y = 54 \text{ km/h}}$$

$$v_x = (95 \text{ km/h})(\cos 35°)$$

$$\boxed{v_x = 78 \text{ km/h}}$$

4. EVALUATE

Because the components of the velocity form a right triangle with the helicopter's actual velocity, the components must satisfy the Pythagorean theorem.

$$v^2 = v_x^2 + v_y^2$$
$$(95)^2 = (78)^2 + (54)^2$$
$$9025 \approx 9000$$

The slight difference is due to rounding.

PRACTICE B

Resolving Vectors

1. How fast must a truck travel to stay beneath an airplane that is moving 105 km/h at an angle of 25° to the ground?
2. What is the magnitude of the vertical component of the velocity of the plane in item 1?
3. A truck drives up a hill with a 15° incline. If the truck has a constant speed of 22 m/s, what are the horizontal and vertical components of the truck's velocity?
4. What are the horizontal and vertical components of a cat's displacement when the cat has climbed 5 m directly up a tree?

ADDING VECTORS THAT ARE NOT PERPENDICULAR

Until this point, the vector-addition problems concerned vectors that are pendicular to one another. However, many objects move in one direction then turn at an angle before continuing their motion.

Suppose that a plane initially travels 5 km at an angle of 35° to the gro then climbs at only 10° relative to the ground for 22 km. How can you d mine the magnitude and direction for the vector denoting the total disp ment of the plane?

Because the original displacement vectors do not form a right triangle can not apply the tangent function or the Pythagorean theorem when ad the original two vectors.

Determining the magnitude and the direction of the resultant ca achieved by resolving each of the plane's displacement vectors into its x a components. Then the components along each axis can be added togeth shown in **Figure 12,** these sums will be the two perpendicular compone the resultant, **d.** The resultant's magnitude can then be found by usin Pythagorean theorem, and its direction can be found by using the invers gent function.

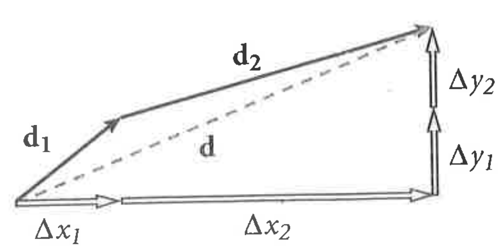


Figure 12
Add the components of the original placement vectors to find two comp that form a right triangle with the re tant vector.

SAMPLE PROBLEM C

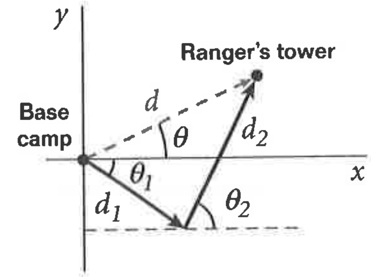
STRATEGY Adding Vectors Algebraically

PROBLEM

A hiker walks 27.0 km from her base camp at 35° south of east. The next day, she walks 41.0 km in a direction 65° north of east and discovers a forest ranger's tower. Find the magnitude and direction of her resultant displacement between the base camp and the tower.

SOLUTION

1. Select a coordinate system. Then sketch and label each vector.



Given: $d_1 = 27.0$ km $\quad \theta_1 = -35°$

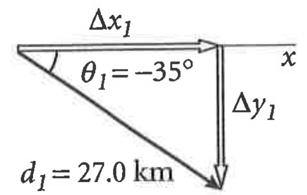
$d_2 = 41.0$ km $\quad \theta_2 = 65°$

Unknown: $d = ?\quad \theta = ?$

> **TIP** *θ_1 is negative, because clockwise angles from the positive x-axis are conventionally considered to be negative.*

2. Find the x and y components of all vectors.

Make a separate sketch of the displacements for each day. Use the cosine and sine functions to find the displacement components.



(a)

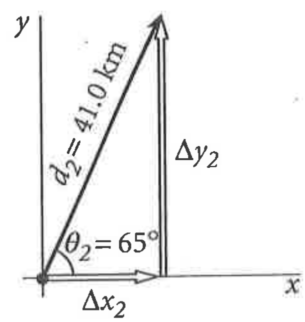
$$\cos\theta = \frac{\Delta x}{d} \qquad \sin\theta = \frac{\Delta y}{d}$$

(a) For day 1: $\Delta x_1 = d_1 \cos\theta_1 = (27.0 \text{ km})\,[\cos(-35°)] = 22 \text{ km}$

$\Delta y_1 = d_1 \sin\theta_1 = (27.0 \text{ km})\,[\sin(-35°)] = -15 \text{ km}$

(b) For day 2: $\Delta x_2 = d_2 \cos\theta_2 = (41.0 \text{ km})\,(\cos 65°) = 17 \text{ km}$

$\Delta y_2 = d_2 \sin\theta_2 = (41.0 \text{ km})\,(\sin 65°) = 37 \text{ km}$



(b)

3. Find the x and y components of the total displacement.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 22 \text{ km} + 17 \text{ km} = 39 \text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -15 \text{ km} + 37 \text{ km} = 22 \text{ km}$$

4. Use the Pythagorean theorem to find the magnitude of the resultant vector.

$$d^2 = (\Delta x_{tot})^2 + (\Delta y_{tot})^2$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(39 \text{ km})^2 + (22 \text{ km})^2} = \boxed{45 \text{ km}}$$

5. Use a suitable trigonometric function to find the angle.

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{22 \text{ km}}{39 \text{ km}}\right) = \boxed{29° \text{ north of east}}$$

PRACTICE C

Adding Vectors Algebraically

1. A football player runs directly down the field for 35 m before turning to the right at an angle of 25° from his original direction and running an additional 15 m before getting tackled. What is the magnitude and direction of the runner's total displacement?

2. A plane travels 2.5 km at an angle of 35° to the ground and then changes direction and travels 5.2 km at an angle of 22° to the ground. What is the magnitude and direction of the plane's total displacement?

3. During a rodeo, a clown runs 8.0 m north, turns 55° north of east, and runs 3.5 m. Then, after waiting for the bull to come near, the clown turns due east and runs 5.0 m to exit the arena. What is the clown's total displacement?

4. An airplane flying parallel to the ground undergoes two consecutive displacements. The first is 75 km 30.0° west of north, and the second is 155 km 60.0° east of north. What is the total displacement of the airplane?

SECTION REVIEW

1. Identify a convenient coordinate system for analyzing each of the following situations:
   - **a.** a dog walking along a sidewalk
   - **b.** an acrobat walking along a high wire
   - **c.** a submarine submerging at an angle of 30° to the horizontal

2. Find the magnitude and direction of the resultant velocity vector for the following perpendicular velocities:
   - **a.** a fish swimming at 3.0 m/s relative to the water across a river that moves at 5.0 m/s
   - **b.** a surfer traveling at 1.0 m/s relative to the water across a wave that is traveling at 6.0 m/s

3. Find the vector components along the directions noted in parentheses.
   - **a.** a car displaced 45° north of east by 10.0 km (north and east)
   - **b.** a duck accelerating away from a hunter at 2.0 m/s^2 at an angle of 35° to the ground (horizontal and vertical)

4. **Critical Thinking** Why do nonperpendicular vectors need to be resolved into components before you can add the vectors together?

Projectile Motion

SECTION 3

SECTION OBJECTIVES

- **Recognize examples of projectile motion.**
- **Describe the path of a projectile as a parabola.**
- **Resolve vectors into their components and apply the kinematic equations to solve problems involving projectile motion.**

TWO-DIMENSIONAL MOTION

In the last section, quantities such as displacement and velocity were shown to be vectors that can be resolved into components. In this section, these components will be used to understand and predict the motion of objects thrown into the air.

Use of components avoids vector multiplication

How can you know the displacement, velocity, and acceleration of a ball at any point in time during its flight? All of the kinematic equations could be rewritten in terms of vector quantities. However, when an object is propelled into the air in a direction other than straight up or down, the velocity, acceleration, and displacement of the object do not all point in the same direction. This makes the vector forms of the equations difficult to solve.

One way to deal with these situations is to avoid using the complicated vector forms of the equations altogether. Instead, apply the technique of resolving vectors into components. Then you can apply the simpler one-dimensional forms of the equations for each component. Finally, you can recombine the components to determine the resultant.

Components simplify projectile motion

When a long jumper approaches his jump, he runs along a straight line, which can be called the x-axis. When he jumps, as shown in **Figure 13,** his velocity has both horizontal and vertical components. Movement in this plane can be depicted by using both the x- and y-axes.

Note that in **Figure 14(b),** a jumper's velocity vector is resolved into its two vector components. This way, the jumper's motion can be analyzed using the kinematic equations applied to one direction at a time.

Figure 13
When the long jumper is in the air, his velocity has both a horizontal and a vertical component.

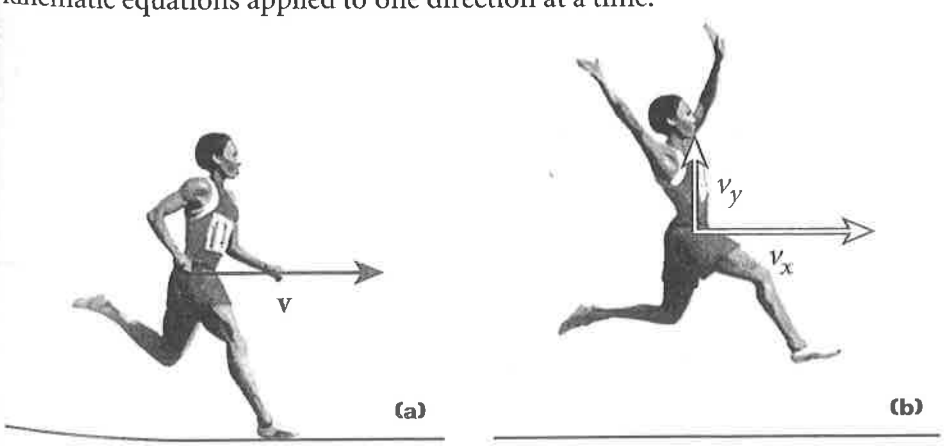


Figure 14
(a) A long jumper's velocity while sprinting along the runway can be represented by a horizontal vector. **(b)** Once the jumper is airborne, the jumper's velocity at any instant can be described by the components of the velocity.

SAMPLE PROBLEM D

Coefficients of Friction

PROBLEM

A 24 kg crate initially at rest on a horizontal floor requires a 75 N horizontal force to set it in motion. Find the coefficient of static friction between the crate and the floor.

SOLUTION

Given: $F_{s,max} = F_{applied} = 75 \text{ N} \quad m = 24 \text{ kg}$

Unknown: $\mu_s = ?$

Use the equation for the coefficient of static friction.

$$\mu_s = \frac{F_{s,max}}{F_n} = \frac{F_{s,max}}{mg}$$

$$\mu_s = \frac{75 \text{ N}}{24 \text{ kg} \times 9.81 \text{ m/s}^2}$$

$$\boxed{\mu_s = 0.32}$$

TIP *Because the crate is on a horizontal surface, the magnitude of the normal force (F_n) equals the crate's weight (mg).*

PRACTICE D

Coefficients of Friction

1. Once the crate in Sample Problem D is in motion, a horizontal force of 53 N keeps the crate moving with a constant velocity. Find μ_k, the coefficient of kinetic friction, between the crate and the floor.

2. A 25 kg chair initially at rest on a horizontal floor requires a 165 N horizontal force to set it in motion. Once the chair is in motion, a 127 N horizontal force keeps it moving at a constant velocity.
   a. Find the coefficient of static friction between the chair and the floor.
   b. Find the coefficient of kinetic friction between the chair and the floor.

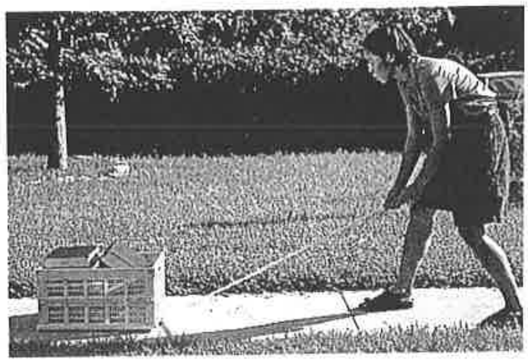
3. A museum curator moves artifacts into place on various different display surfaces. Use the values in **Table 2** to find $F_{s,max}$ and F_k for the following situations:
   a. moving a 145 kg aluminum sculpture across a horizontal steel platform
   b. pulling a 15 kg steel sword across a horizontal steel shield
   c. pushing a 250 kg wood bed on a horizontal wood floor
   d. sliding a 0.55 kg glass amulet on a horizontal glass display case

SAMPLE PROBLEM E

Overcoming Friction

PROBLEM

A student attaches a rope to a 20.0 kg box of books. He pulls with a force of 90.0 N at an angle of 30.0° with the horizontal. The coefficient of kinetic friction between the box and the sidewalk is 0.500. Find the acceleration of the box.



SOLUTION

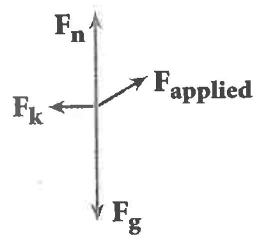
1. DEFINE

Given: $m = 20.0$ kg $\mu_k = 0.500$

$\mathbf{F}_{\mathbf{applied}} = 90.0$ N at $\theta = 30.0°$

Unknown: $\mathbf{a} = ?$

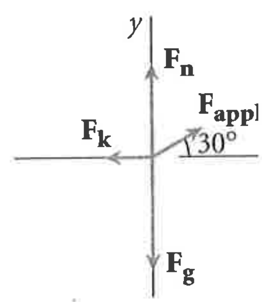
Diagram:



2. PLAN

Choose a convenient coordinate system, and find the *x* and *y* components of all forces.

The diagram at right shows the most convenient coordinate system, because the only force to resolve into components is $\mathbf{F}_{\mathbf{applied}}$.



$F_{applied,y} = (90.0 \text{ N})(\sin 30.0°) = 45.0$ N (upward)

$F_{applied,x} = (90.0 \text{ N})(\cos 30.0°) = 77.9$ N (to the right)

Choose an equation or situation:

A. Find the normal force, F_n, by applying the condition of equilibrium in the vertical direction: $\Sigma F_y = 0$.

B. Calculate the force of kinetic friction on the box: $F_k = \mu_k F_n$.

C. Apply Newton's second law along the horizontal direction to find the acceleration of the box: $\Sigma F_x = ma_x$.

3. CALCULATE

Substitute the values into the equations and solve:

A. To apply the condition of equilibrium in the vertical direction, you need to account for all of the forces in the *y* direction: F_g, F_n, and $F_{applied,y}$. You know $F_{applied,y}$ and can use the box's mass to find F_g.

$F_{applied,y} = 45.0$ N

$F_g = (20.0 \text{ kg})(9.81 \text{ m/s}^2) = 196$ N

Next, apply the equilibrium condition, $\Sigma F_y = 0$, and solve for F_n.

$$\Sigma F_y = F_n + F_{applied,y} - F_g = 0$$

$$F_n + 45.0 \text{ N} - 196 \text{ N} = 0$$

$$F_n = -45.0 \text{ N} + 196 \text{ N} = 151 \text{ N}$$

TIP *Remember to pay attention to the direction of forces. Here, F_g is subtracted from F_n and $F_{applied,y}$ because F_g is directed downward.*

B. Use the normal force to find the force of kinetic friction.

$$F_k = \mu_k F_n = (0.500)(151 \text{ N}) = 75.5 \text{ N}$$

C. Use Newton's second law to determine the horizontal acceleration.

TIP *$\mathbf{F_k}$ is directed toward the left, opposite the direction of $\mathbf{F_{applied,x}}$. As a result, when you find the sum of the forces in the x direction, you need to subtract F_k from $F_{applied,x}$.*

$$\Sigma F_x = F_{applied,x} - F_k = ma_x$$

$$a_x = \frac{F_{applied,x} - F_k}{m} = \frac{77.9 \text{ N} - 75.5 \text{ N}}{20.0 \text{ kg}} = \frac{2.4 \text{ N}}{20.0 \text{ kg}} = \frac{2.4 \text{ kg}\bullet\text{m/s}^2}{20.0 \text{ kg}}$$

$$\mathbf{a} = 0.12 \text{ m/s}^2 \text{ to the right}$$

4. EVALUATE The normal force is not equal in magnitude to the weight because the y component of the student's pull on the rope helps support the box.

PRACTICE E

Overcoming Friction

1. A student pulls on a rope attached to a box of books and moves the box down the hall. The student pulls with a force of 185 N at an angle of 25.0° above the horizontal. The box has a mass of 35.0 kg, and μ_k between the box and the floor is 0.27. Find the acceleration of the box.
2. The student in item 1 moves the box up a ramp inclined at 12° with the horizontal. If the box starts from rest at the bottom of the ramp and is pulled at an angle of 25.0° with respect to the incline and with the same 185 N force, what is the acceleration up the ramp? Assume that $\mu_k = 0.27$.
3. A 75 kg box slides down a 25.0° ramp with an acceleration of 3.60 m/s^2.
   a. Find μ_k between the box and the ramp.
   b. What acceleration would a 175 kg box have on this ramp?
4. A box of books weighing 325 N moves at a constant velocity across the floor when the box is pushed with a force of 425 N exerted downward at an angle of 35.2° below the horizontal. Find μ_k between the box and the floor.